The Complexity of Synthesizing Uniform Strategies

Bastien Maubert
IRISA
Université de Rennes 1
Rennes, France
bastien.maubert@irisa.fr

Sophie Pinchinat
IRISA
Université de Rennes 1
Rennes, France
sophie.pinchinat@irisa.fr

Laura Bozzelli
Facultad de Informática
UPM
Madrid, Spain
laura.bozzelli@fi.upm.es

We investigate *uniformity properties* of strategies. These properties involve sets of plays in order to express useful constraints on strategies that are not μ-calculus definable. Typically, we can state that a strategy is observation-based. We propose a formal language to specify uniformity properties, interpreted over two-player turn-based arenas equipped with a binary relation between plays. This way, we capture *e.g.* games with winning conditions expressible in epistemic temporal logic, whose underlying equivalence relation between plays reflects the observational capabilities of agents (for example, synchronous perfect recall). Our framework naturally generalizes many other situations from the literature. We establish that the problem of synthesizing strategies under uniformity constraints based on regular binary relations between plays is non-elementary complete.

1 Introduction

In extensive infinite duration games, the arena is represented as a graph whose vertices denote positions of players and whose paths denote plays. In this context, a strategy of a player is a mapping prescribing to this player which next position to select provided she has to make a choice at this current point of the play. As mathematical objects, strategies can be seen as infinite trees obtained by pruning the infinite unfolding of the arena according to the selection prescribed by this strategy; outcomes of a strategy are therefore the branches of the trees.

Strategies of players are not arbitrary in general, since players aim at achieving some objectives. Infinite-duration game models have been intensively studied for their applications in computer science [3] and logic [13]. First, infinite-duration games provide a natural abstraction of computing systems' non-terminating interaction [2] (think of a communication protocol between a printer and its users, or control systems). Second, infinite-duration games naturally occur as a tool to handle logical systems for the specification of non-terminating behaviors, such as for the propositional μ-calculus [10], leading to a powerful theory of automata, logics and infinite games [13] and to the development of algorithms for the automatic verification ("model-checking") and synthesis of hardware and software systems. In both cases, outcomes of strategies are submitted to ω-regular conditions representing some desirable property of a system.

Additionally, the cross fertilization of multi-agent systems and distributed systems theories has led to equip logical systems with additional modalities, such as epistemic ones, to capture uncertainty [27, 21, 11, 24, 20, 15], and more recently, these logical systems have been adapted to game models in order to reason about knowledge, time and strategies [17, 19, 9]. The whole picture then becomes intricate, mainly because time and knowledge are essentially orthogonal, yielding a complex theoretical universe to reason about. In order to understand to which extent knowledge and time are orthogonal, the angle of view where strategies are infinite trees is helpful: Time is about the *vertical* dimension of the trees as it relates to the ordering of encountered positions along plays (branches) and to the branching in the tree.

Submitted to:
SR 2013

On the contrary, Knowledge is about the *horizontal* dimension, as it relates plays carrying, e.g., the same information.

As far as we know, this horizontal dimension, although extensively studied when interpreted as knowledge or observation [4, 17, 19, 8, 1, 9], has not been addressed in its generality. In this paper, we aim at providing a unified setting to handle it. We introduce the generic notion of *uniformity properties* and associated so-called *uniform strategies* (those satisfying uniformity properties). Some notions of "uniform" strategies have already been used, e.g., in the setting of strategic logics [29, 5, 19] and in the evaluation game of Dependence Logic [28], which both fall into the general framework we present here.

We use a simple framework with two-player turn-based arenas and where information lies in positions, but the approach can be extended to other settings. Additionally, although uniformity properties can be described in a set-theoretic framework, we propose the logical formalism $\mathbb{R}\mathsf{LTL}$ which can be exploited to address fundamental automated techniques such as the verification of uniformity properties and the synthesis of uniform strategies – arbitrary uniformity properties are in general hopeless for automation. The formalism we use combines the Linear-time Temporal Logic LTL [12] and a new modality $\mathbb{R}$ (for "for all related plays"), the semantics of which is given by a binary relation between plays. Modality $\mathbb{R}$ generalizes the knowledge operator "K" of [15] for the epistemic relations of agents in Interpreted Systems. The semantic binary relations between plays are very little constrained: they are not necessarily equivalences, to capture, *e.g.* plausibility (pre)orders one finds in doxastic logic [16], neither are they knowledge-based, to capture particular strategies in games where epistemic aspects are irrelevant. Formulas of the logic are interpreted over outcomes of a strategy. The $\mathbb{R}$ modality allows to universally quantify over all plays that are in relation with the current play. Distinguishing between the universal quantification over all plays in the game and the universal quantification over all the outcomes in the strategy tree yields two kinds of uniform strategies: the *fully-uniform strategies* and the *strictly-uniform strategies*.

As extensively demonstrated in [22], uniform properties turn out to be many in the literature: they occur in games with imperfect information, in games with opacity conditions and more generally with epistemic conditions, as non-interference properties of computing systems, as diagnosability of discrete-event systems, in the game semantics of Dependence Logic.

We investigate the automated synthesis of fully-uniform strategies, for the case of finite arenas and binary relations between plays that are rational in the sense of [6]. Incidentally, all binary relations that are involved in the relevant literature seem to follow this restriction. In this context, two problems can be addressed: the *fully-uniform strategy problem* and the *strictly-uniform strategy problem*, which essentially can be formulated as "given a finite arena, a finite state transducer describing a binary relation between plays, and a formula expressing a uniformity property, does there exist a fully-uniform (resp. strictly-uniform) strategy for Player 1?". From [22], the fully-uniform strategy problem is decidable but non-elementary – since then we have established that it is non-elementary hard. The algorithm involves an iterated non-trivial powerset construction from the arena and the finite state transducer which enables to eliminate innermost $\mathbb{R}$ modalities. Hence, the required number of iterations matches the maximum number of nested $\mathbb{R}$ modalities of the formula expressing the uniformity property. As expected, each powerset construction is computed in exponential time. This procedure amounts to solving an ultimate LTL game, for which a strategy can be synthesized [25] and traced back as a solution in the original problem. The decidability of the strictly-uniform strategy problem is an open question.

The rest of the paper is organized in five sections. In Section 2, we present the standard material two-player turn-based arenas. We set up the framework and define uniform strategies in Section 3, and we illustrate the notion with two examples in Section 4. Finally in Section 5, we give tight complexity bounds for the fully-uniform strategy problem, and we discuss future work in Section 6.

2 Preliminaries

We consider two-player turn-based games that are played on graphs with vertices labelled with propositions. These propositions represent the relevant information for the uniformity properties one wants to state. From now on and for the rest of the paper, we let AP be an infinite set of *atomic propositions*.

An *arena* is a structure $\mathcal{G} = (V, E, v_0, \ell)$ where $V = V_1 \uplus V_2$ is the set of *positions*, partitioned between positions of Player 1 (V_1) and those of Player 2 (V_2), $E \subseteq (V_1 \times V_2) \cup (V_2 \times V_1)$ is the set of *edges*, $v_0 \in V$ is the *initial position* and $\ell : V \to \mathscr{P}(AP)$ is a *valuation function*, mapping each position to the finite set of atomic propositions that hold in this position. $Plays_*$ and $Plays_\omega$ are, respectively, the set of finite and infinite *plays*. For an infinite play $\pi = v_0 v_1 \ldots$ and $i \in \mathbb{N}$, $\pi[i] := v_i$ and $\pi[0,i] := v_0 \ldots v_i$. For a finite play $\rho = v_0 v_1 \ldots v_n$, $\mathrm{last}(\rho) = v_n$.

A *strategy* for Player 1 is a partial function $\sigma : Plays_* \to V$ that maps a finite play ending in V_1 to the next position to play. Let σ be a strategy for Player 1. We say that a play $\pi \in Plays_\omega$ is *induced by* σ if for all $i \geq 0$ such that $\pi[i] \in V_1$, $\pi[i+1] = \sigma(\pi[0,i])$, and the *outcome of* σ, noted $\mathrm{Out}(\sigma) \subseteq Plays_\omega$, is the set of all infinite plays that are induced by σ. Definitions are similar for Player 2's strategies.

3 Uniform strategies

We define the formal language $\mathbb{R}$LTL to specify uniformity properties. This language enables to express properties of the dynamics of plays, and resembles the Linear Temporal Logic (LTL) [12]. However, while LTL formulas are evaluated on individual plays (paths), we want here to express properties on "bundles" of plays. To this aim, we equip arenas with a binary relation between finite plays, and we enrich the logic with a modality $\mathbb{R}$ that quantifies over related plays, the intended meaning of "$\mathbb{R}\varphi$ holds in ρ" being "φ holds in every play related to ρ".

The syntax of $\mathbb{R}$LTL is similar to that of linear temporal logic with knowledge [15]. However, we use $\mathbb{R}$ instead of the usual knowledge operator K to emphasize that it need not be interpreted in terms of knowledge in general, but merely as a way to state properties of bundles of plays. The syntax is:

$$\varphi, \psi ::= p \mid \neg\varphi \mid \varphi \wedge \psi \mid \bigcirc\varphi \mid \varphi \mathbf{U} \psi \mid \mathbb{R}\varphi \qquad p \in AP$$

Consider an arena $\mathcal{G} = (V, E, v_0, \ell)$ and a rational relation $\rightsquigarrow \subseteq Plays_* \times Plays_*$. A formula φ of $\mathbb{R}$LTL is evaluated at some point $i \in \mathbb{N}$ of an infinite play $\pi \in Plays_\omega$, within a *universe* $\Pi \subseteq Plays_\omega$. The semantics is given by induction over formulas.

$$\begin{array}{ll}
\Pi, \pi, i \models p \;\text{ if }\; p \in \ell(\pi[i]) & \Pi, \pi, i \models \neg\varphi \;\text{ if }\; \Pi, \pi, i \not\models \varphi \\
\Pi, \pi, i \models \varphi \wedge \psi \;\text{ if }\; \Pi, \pi, i \models \varphi \text{ and } \Pi, \pi, i \models \psi & \Pi, \pi, i \models \bigcirc\varphi \;\text{ if }\; \Pi, \pi, i+1 \models \varphi \\
\multicolumn{2}{l}{\Pi, \pi, i \models \varphi \mathbf{U} \psi \;\text{ if there is } j \geq i \text{ such that } \Pi, \pi, j \models \psi \text{ and for all } i \leq k < j,\ \Pi, \pi, k \models \varphi} \\
\multicolumn{2}{l}{\Pi, \pi, i \models \mathbb{R}\varphi \;\text{ if for all } \pi' \in \Pi, j \in \mathbb{N} \text{ such that } \pi[0,i] \rightsquigarrow \pi'[0,j],\ \Pi, \pi', j \models \varphi}
\end{array}$$

From this semantics, we derive two notions of uniform strategies, which differ only in the universe the $\mathbb{R}$ modality quantifies over: $\mathrm{Out}(\sigma)$ or $Plays_\omega$ (with the latter, related plays not induced by the strategy also count). The motivation for these two definitions is clear from [22] where many examples from the literature are given.

Definition 1 *Let $\mathcal{G}$ be an arena, $\rightsquigarrow$ be a rational relation and φ be an $\mathbb{R}$LTL formula. A strategy σ is:*
$(\rightsquigarrow, \varphi)$-strictly-uniform *if for all $\pi \in Out(\sigma)$, $Out(\sigma), \pi, 0 \models \varphi$,*
$(\rightsquigarrow, \varphi)$-fully-uniform *if for all $\pi \in Out(\sigma)$, $Plays_\omega, \pi, 0 \models \varphi$.*

4 Concrete examples

In this section we illustrate our notions of strictly and fully uniform strategies defined in the previous section with the examples of observation-based strategies in games with imperfect information, and games with opacity condition.

4.1 Observation-based strategies

Games with imperfect information, in general, are games in which some of the players do not know exactly what is the current position of the game. Poker is an example of imperfect-information game: one does not know which cards her opponents have in hands. One important aspect of imperfect-information games is that not every strategy is "playable". Indeed, a player cannot plan to play differently in situations that she is unable to distinguish. This is why players are required to use strategies that select moves uniformly over observationally equivalent situations. This kind of strategies is sometimes called *uniform strategies* in the community of strategic logics ([29, 5, 19]), or *observation-based strategies* in the community of computer-science oriented game theory ([8]). In fact, all the additional complexity of solving imperfect-information games, compared to perfect-information ones, lies in this constraint put on strategies.

We show that the notion of observation-based strategy, and hence the essence of games with imperfect information, can be easily embedded in our notion of uniform strategy. In two-player imperfect-information games as studied for example in [26, 8, 7], Player 1 only partially observes the positions of the game, such that some positions are indistinguishable to her, while Player 2 has perfect information (the asymmetry is due to the focus being on the existence of strategies for Player 1). Arenas are labelled directed graphs together with a finite set of *actions* Act, and in each round, if the position is a node v, Player 1 chooses an available action a, and Player 2 chooses a next position v' reachable from v through an a-labelled edge.

We equivalently define this framework in a manner that fits our setting by putting Player 1's actions inside the positions. We have two kinds of positions, of the form v and of the form (v,a). In a position v, when she chooses an action a, Player 1 actually moves to position (v,a), then Player 2 moves from (v,a) to some v'. So an imperfect-information game arena is a structure $\mathscr{G}_{imp} = (\mathscr{G}, \sim)$ where $\mathscr{G} = (V, E, v_0, \ell)$ is a two-player game arena with positions in V_1 of the form v and positions in V_2 of the form (v,a). We require that $vE(v',a)$ implies $v = v'$, and $v_0 \in V_1$. For a position $(v,a) \in V_2$, we note $(v,a).act := a$. We assume that $p_1 \in AP$, and for every action a in Act, $p_a \in AP$. p_1 holds in positions belonging to Player 1, and p_a holds in positions of Player 2 where the last action chosen by Player 1 is a: $\ell(v) = \{p_1\}$ for $v \in V_1$, $\ell(v,a) = \{p_a\}$ for $(v,a) \in V_2$. Finally, $\sim \subseteq V_1^2$ is an observational equivalence relation on positions, that relates positions indistinguishable for Player 1. We define its extension $\simeq$ to finite plays: $v_0(v_0,a_1)v_1 \dots (v_{n-1},a_n)v_n \simeq v_0(v_0,a'_1)v'_1 \dots (v'_{n-1},a'_n)v'_n$ if for all $i > 0$, $v_i \sim v'_i$ and $a_i = a'_i$.

We add the classic requirement that the same actions must be available in indistinguishable positions: for all $v, v' \in V_1$, if $v \sim v'$ then $vE(v,a)$ if, and only if, $v'E(v',a)$. In other words, if Player 1 has different options, she can distinguish the positions.

Definition 2 *A strategy σ for Player 1 is* observation-based *if for all $\rho, \rho' \in v_0(V_2V_1)^*$, $\rho \approx \rho'$ implies $\sigma(\rho).act = \sigma(\rho').act$.*

We define the formula

$$\mathtt{SameAct} := \mathbf{G}(p_1 \to \bigvee_{a \in Act} \mathbb{R} \bigcirc p_a)$$

which, informally, expresses that whenever it is Player 1's turn to play, there is an action a that is played in every equivalent finite play.

Proposition 1 *A strategy σ for Player 1 is observation-based iff it is $(\approx, \texttt{SameAct})$-strictly-uniform.*

Here we have to make use of the notion of strict uniformity, and not the full uniformity. Indeed, after a finite play $\pi[0,i]$ ending in V_1, we want to enforce that in all equivalent prefixes of infinite plays *that conform to the strategy considered*, Player 1 plays the same action. It would obviously make no sense to enforce the same on equivalent prefixes of every possible play in the game, which encompass all possible behaviours of Player 1.

Notice that in order to embed the case of players with different memory abilities, *e.g.* imperfect-recall, one would just have to replace $\approx$ with the appropriate relation.

For the moment we have not mentioned any winning condition. For a strategy, being $(\approx, \texttt{SameAct})$-strictly-uniform only characterizes that it is "playable" for a player with imperfect information, but it does not characterize the outcome of this strategy. However, if one considers a game with imperfect information in which the winning condition for Player 1 is an LTL formula φ, then the set of $(\approx, \texttt{SameAct} \wedge \varphi)$-strictly-uniform strategy is exactly the set of winning observation-based strategy.

When talking about knowledge and strategic abilities, the question of *objective* vs *subjective* ability should be raised (see [18]). The difference is basically whether a strategy is defined only on "concrete" plays, starting from the initial position, or if it has to be defined on all "plays" starting from any position the player confuses with the initial one. In the setting presented here, the initial position is part of the description of the arena, hence players are assumed to know it and all plays considered start from this position. But in order to model in this setting the case of Player 1 not knowing the initial position, one could add a fresh artificial initial position v_0', from which no matter the action Player 1 chooses, Player 2 can move to any position that Player 1 confuses with v_0. Then, for a winning condition $\varphi \in \mathsf{LTL}$, the existence of an observation-based winning strategy for Player 1 from v_0 (resp. v_0') would denote objective (resp. subjective) ability to enforce φ.

4.2 Games with opacity condition

Games with opacity condition, studied in [23], are based on two-player imperfect-information arenas, with Alice having perfect information as opposed to Bob who partially observes positions. In such games, some positions are "secret" as they reveal a critical information that Bob aims at discovering. We are interested in Alice's ability to prevent Bob from "knowing" the secret, in the epistemic sense.

More formally, assume that a proposition $p_S \in AP$ represents the secret. Let $\mathscr{G}_{inf} = (\mathscr{G}, \sim)$ be an imperfect-information arena as described in Section 4.1, with a distinguished set of positions $S \subseteq V_1$ that denotes the secret. Bob is Player 1 as he has imperfect information, and Alice is Player 2. Letting $\mathscr{G} = (V, E, v_I, \ell)$, we require that $\ell^{-1}(\{p_S\}) = S$ (positions labeled by p_S are exactly positions $v \in S$). For a finite play ρ with $last(\rho) \in V_1$, Bob's *information set* or *knowledge* after ρ is $I(\rho) := \{last(\rho') \mid \rho' \in Plays_*, \rho \approx \rho'\}$. It is the set of all the positions he considers possible after observing ρ. An infinite play is winning for Bob if there exists a finite prefix ρ of this play whose information set is contained in S, *i.e.* $I(\rho) \subseteq S$, otherwise Alice wins. It can easily be shown that:

Proposition 2 *A strategy σ for Alice is winning if, and only if, σ is $(\approx, \mathbf{G}\neg \mathbb{R} p_S)$-fully-uniform.*

Here we are interested in Alice's strategies and Bob's knowledge. Since Bob only partially observes what Alice is playing, some plays that are not brought about by Alice's strategy are considered possible by Bob. Full uniformity is therefore the right notion to capture correctly Bob's knowledge.

Here again, to model different memory and observational abilities of Bob, one can use the appropriate binary relation, provided it is rational. Also, notice that though we chose to illustrate our framework with opacity aspects, any winning condition that is expressible by a formula of the epistemic linear temporal logic with one knowledge operator would fit in our setting.

5 Synthesizing fully-uniform strategies

In this section, we investigate the complexity of synthesizing a fully-uniform strategy. We first consider the associated decision problem, called here the *fully-uniform strategy problem*: given a uniform property $\varphi \in \mathbb{R}\mathsf{LTL}$, a finite arena $\mathcal{G} = (V, E, v_0, \ell)$, and a finite state transducer T over alphabet V representing a rational binary relation between plays (see [6]), does there exist a $([T], \varphi)$-fully-uniform strategy in $\mathcal{G}$, where $[T]$ is the binary relation denoted by T.

Definition 3 *For a formula $\varphi \in \mathbb{R}\mathsf{LTL}$, the* $\mathbb{R}$-depth *of φ, written $d_{\mathbb{R}}(\varphi)$, is the maximum number of nested $\mathbb{R}$ modalities in φ. For each $k \in \mathbb{N}$, we let $\mathbb{R}\mathsf{LTL}_k := \{\varphi \in \mathbb{R}\mathsf{LTL} \mid d_{\mathbb{R}}(\varphi) = k\}$.*

Theorem 3 *The fully-uniform strategy problem for formulas ranging over $\bigcup_{k \le n} \mathbb{R}\mathsf{LTL}_k$ is n-*EXPTIME*-complete for $n > 2$, and* 2EXPTIME*-complete for $n \le 2$.*

The proof for the upper bounds in Theorem 3 can be found in [22], in which we devise a decision procedure based on a powerset construction which simulates the execution of the transducer along plays in the arena, enabling the computation of information sets. Dealing with information sets enables us to perform $\mathbb{R}$-modalities elimination, yielding a reduction of the initial problem to solving some LTL game. The procedure is however non-elementary since it requires one powerset construction per nesting of $\mathbb{R}$-modalities. The proof for the matching lower bounds is a direct reduction from the word problem for $\exp[n]$-space bounded alternating Turing Machines, which is $(n+1)$-EXPTIME complete. Due to lack of space, it is omitted here.

Corollary 4 *The fully-uniform strategy problem is non-elementary complete.*

Regarding the synthesis problem, the procedure of [25] for solving the terminal LTL game in the decision procedure of Theorem 3 is an effective construction of a winning strategy when it exists. This strategy provides a fully-uniform strategy of the initial game, by means of a transducer mapping plays of the initial game to plays in the terminal game. This transducer itself is straightforwardly built from the arena of the last game itself.

6 Discussion

We are currently working on sufficient conditions on the binary relation between plays to render the fully-uniform strategy synthesis problem elementary. It appears that being an equivalence relation is not enough, but if moreover the relation verifies a weak form of *no learning* property (see [14]), the problem seems to be elementary. Concerning the strictly-uniform strategy problem, we conjecture undecidability in general, but we are investigating interesting subclasses of rational relations that make the problem decidable.

It would then be interesting to extend the language to the case of n modalities $\mathbb{R}_i$ with n relations $\rightsquigarrow_i$. Also, the difference between the fully-uniform semantics and the strictly-uniform one could be at the level of modalities instead of the decision problems level. In Section 4.1 we have seen that uniformity properties can represent *uniformity constraints* on the set of elegible strategies, and in Section 4.2 we have seen how they can represent *epistemic winning conditions*. However, while some properties require strict uniformity, others require full uniformity. Allowing to use both kinds of modalities in a formula would enable, for example, to express that a strategy must both be winning for some condition on the opponent's knowledge (with a fully-uniform modality, see Section 4.2), and to be observation based for the player considered (with a strictly-uniform modality). A formula of the following kind could be used for a variant of games with opacity condition where Alice would also have imperfect information (note that the arenas should be modified, and we assume that p_2 would mark positions where Alice has to choose an action):

$$\varphi := \underbrace{\mathbf{G}(p_2 \to \bigvee_{a \in Act} \mathbb{R}^{strictly}_{Alice} \bigcirc p_a)}_{\text{Observation-based constraint}} \quad \wedge \quad \underbrace{\mathbf{G}\neg\mathbb{R}^{fully}_{Bob}\, p_S}_{\text{Winning condition}}$$

In a next step, we would like to consider how our framework adapts if we take as base language the one of Alternating-time Temporal Logic [2] instead of LTL, so as to obtain an Alternating-time Temporal Epistemic Logic-like language. It would enable us to express the existence of uniform strategies directly in the logic, and not only at the level of decision problems as it is the case for now. This step will require to pass from the two-player turn-based arenas considered so far to multiplayer concurrent game structures, that are ATL models, but the definitions should adapt without difficulties. However we should be cautious in generalizing these notions as undecidability will easily be attained.

References

[1] R. Alur, P. Černỳ & S. Chaudhuri (2007): *Model checking on trees with path equivalences*. Tools and Algorithms for the Construction and Analysis of Systems, pp. 664–678.

[2] R. Alur, T.A. Henzinger & O. Kupferman (2002): *Alternating-time temporal logic*. Journal of the ACM (JACM) 49(5), pp. 672–713.

[3] K.R. Apt & E. Grädel (2011): *Lectures in Game Theory for Computer Scientists*. Cambridge University Press.

[4] A. Arnold, A. Vincent & I. Walukiewicz (2003): *Games for synthesis of controllers with partial observation*. Theoretical Computer Science 1(303), pp. 7–34. Available at `http://www.labri.fr/Perso/~igw/Papers/igw-synthesis.ps`.

[5] J. Benthem (2005): *The Epistemic Logic of IF Games*. The Philosophy of Jaakko Hintikka 30.

[6] J. Berstel (1979): *Transductions and context-free languages*. 4, Teubner Stuttgart.

[7] D. Berwanger & L. Doyen (2008): *On the power of imperfect information*. In: Proc. of FSTTCS, Citeseer, pp. 73–82. Available at `http://drops.dagstuhl.de/opus/volltexte/2008/1742`.

[8] K. Chatterjee, L. Doyen, T. Henzinger & J.F. Raskin (2006): *Algorithms for omega-regular games with imperfect information*. In: Computer Science Logic, Springer, pp. 287–302.

[9] C. Dima, C. Enea & D. Guelev (2010): *Model-Checking an Alternating-time Temporal Logic with Knowledge, Imperfect Information, Perfect Recall and Communicating Coalitions*. Electronic Proceedings in Theoretical Computer Science 25.

[10] E. A. Emerson & C. S. Jutla (1991): *Tree Automata, Mu-Calculus and Determinacy*. In: Proceedings 32nd Annual IEEE Symp. on Foundations of Computer Science, FOCS'91, San Jose, Puerto Rico, 1–4 Oct 1991, IEEE Computer Society Press, Los Alamitos, California, pp. 368–377.

[11] R. Fagin, J.Y. Halpern & M.Y. Vardi (1991): *A model-theoretic analysis of knowledge*. Journal of the ACM (JACM) 38(2), pp. 382–428.

[12] D. Gabbay, A. Pnueli, S. Shelah & J. Stavi (1980): *On the Temporal Analysis of Fairness*. In: Proc. 7th ACM Symp. Principles of Distributed Computing, Toronto, Ontario, Canada, pp. 163–173.

[13] E. Grädel, W. Thomas & T. Wilke, editors (2002): *Automata, Logics, and Infinite Games: A Guide to Current Research [outcome of a Dagstuhl seminar, February 2001]*. Lecture Notes in Computer Science 2500, Springer.

[14] J.Y. HALPERN, R. VAN DER MEYDEN & M.Y. VARDI (2004): *Complete axiomatizations for reasoning about knowledge and time*. SIAM journal on computing 33(3), pp. 674–703.

[15] J.Y. Halpern & M.Y. Vardi (1989): *The complexity of reasoning about knowledge and time. 1. Lower bounds*. Journal of Computer and System Sciences 38(1), pp. 195–237, doi:10.1145/12130.12161.

[16] J. Hintikka (1962): *Knowledge and belief*. 13, Cornell University Press Ithaca.

[17] W. van der Hoek & M. Wooldridge (2003): *Cooperation, knowledge, and time: Alternating-time temporal epistemic logic and its applications*. Studia Logica 75(1), pp. 125–157.

[18] W. Jamroga & N. Bulling (2011): *Comparing variants of strategic ability*. In: Proceedings of the Twenty-Second international joint conference on Artificial Intelligence-Volume Volume One, AAAI Press, pp. 252–257.

[19] W. Jamroga & W.V.D. Hoek (2004): *Agents that know how to play*. Fundamenta Informaticae 63(2-3), pp. 185–220.

[20] R.E. Ladner & J.H. Reif (1986): *The logic of distributed protocols: preliminary report*. In: Proceedings of the 1986 Conference on Theoretical aspects of reasoning about knowledge, Morgan Kaufmann Publishers Inc., pp. 207–222.

[21] D. Lehmann (1984): *Knowledge, common knowledge and related puzzles (Extended Summary)*. In: Proceedings of the third annual ACM symposium on Principles of distributed computing, ACM, pp. 62–67.

[22] Bastien Maubert & Sophie Pinchinat (2012): *Uniform Strategies*. Rapport de recherche RR-8144, INRIA. Available at `http://hal.inria.fr/hal-00760370`.

[23] Bastien Maubert, Sophie Pinchinat & Laura Bozzelli (2011): *Opacity Issues in Games with Imperfect Information*. In Giovanna D'Agostino & Salvatore La Torre, editors: Proceedings Second International Symposium on Games, Automata, Logics and Formal Verification, Minori, Italy, 15-17th June 2011, Electronic Proceedings in Theoretical Computer Science 54, Open Publishing Association, pp. 87–101, doi:10.4204/EPTCS.54.7.

[24] R. Parikh & R. Ramanujam (1985): *Distributed processes and the logic of knowledge*. Logics of Programs, pp. 256–268.

[25] A. Pnueli & R. Rosner (1989): *On the Synthesis of an Asynchronous Reactive Module*. In: Proc. 16th Int. Coll. on Automata, Languages and Programming, ICALP'89, Stresa, Italy, LNCS 372, Springer-Verlag, pp. 652–671.

[26] J.H. Reif (1984): *The complexity of two-player games of incomplete information*. Journal of computer and system sciences 29(2), pp. 274–301, doi:10.1016/0022-0000(84)90034-5.

[27] JM Sato (1977): *A study of Kripke style methods for some modal logic by Gentzen's sequential method*. Technical Report, Technical report, Publication Research Institute for Mathematical Science.

[28] J. Väänänen (2007): *Dependence Logic*.

[29] J. Van Benthem (2001): *Games in Dynamic-Epistemic Logic*. Bulletin of Economic Research 53(4), pp. 219–248.